## Research on autonomous learning strategy of higher mathematics

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**Abstract:** In view of the problems existing in the teaching and learning of higher mathematics, this paper mainly discusses the improvement of teaching methods, the all-round provision of teaching resources, the change of teaching forms and the teaching management, and discusses how to construct the autonomous learning strategy of higher mathematics.

#### 1. Introduction

Promoting the improvement of students' independent learning degree not only depends on students' own subject consciousness and independent learning ability, but also depends on teachers' teaching concept and overall grasp of teaching content and teaching methods. According to the characteristics of higher mathematics teaching, students could truly experience the fun of higher mathematics course through practical problem analysis, reasoning, deduction and induction, which is beneficial to stimulate students' interest in further learning and enhance their awareness and ability of independent learning.

In this paper, through the investigation and analysis of current situation and influencing factors of students' autonomous learning of higher mathematics in Nanjing Xiaozhuang University, the countermeasures of cultivating students' autonomous learning ability of higher mathematics are put forward.

#### 2. Stimulating learning motivation

Learning motivation is the premise and foundation of learning activities. Only with a strong learning motivation, we can take the initiative to study higher mathematics and achieve good learning results. Only by stimulating students' learning motivation and cultivating their interests in seeking knowledge, we can make students' learning have lasting motivation and change "wanting me to learn" into "I want to learn". Interest in higher mathematics learning is an important basis for students' autonomous learning and a catalyst to promote their autonomous learning.

The self-stimulation of learning motivation is influenced by learning goal orientation, learning interest, self-efficacy, attribution and other factors. Therefore, teachers can stimulate learning motivation by helping students set scientific learning objectives for advanced mathematics.

Goal setting has the function of orientation, adjustment, maintenance and evaluation for individual's independent learning. Therefore, whether students can set suiTable mathematics learning goals for themselves will have an important impact on their independent learning of higher mathematics. In the process of teaching, teachers should transform teaching objectives into students' learning objectives, and help students set scientific learning objectives according to the actual situation of students.

## 2.1 Setting short-term and specific learning goals

American psychologists Hogue and Glee believed that long-term goals should be combined with short-term goals. Long-term goals should be decomposed into short-term and specific sub-goals.

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Compared with long-term goals, short-term goals are easier to achieve and enable students to experience success more quickly. In teaching, teachers should guide students to decompose long-term goals into short-term goals. In addition, because of the Abstractness of mathematical research objects, some problems are really difficult to explain with practical background. Therefore, in the mathematics classroom teaching in self-learning, students should be taught to decompose Abstract and difficult learning objectives into relatively concrete and simple ones. For example, there is function  $\mathbf{y} = \arctan\ln(3x - 1)$ , we need to solve  $\mathbf{y}$ . First, we let  $\mathbf{y} = \arctan\mathbf{u}, \mathbf{u} = \ln\mathbf{v}, \mathbf{v} = 3\mathbf{x} - 1$ , transform complex functions into relatively simple basic elementary functions. And then, we use derivation formula solve  $\frac{d\mathbf{y}}{d\mathbf{u}}, \frac{d\mathbf{u}}{d\mathbf{v}}, \frac{d\mathbf{v}}{d\mathbf{x}}$ , and use compound

function derivation law solve  $y' = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{3}{(3x-1)[1+ln^2 (3x-1)]}$ . By completing tasks step by

step, students can experience the progress of their learning, enhance their sense of self-efficacy, and stimulate their desire for independent learning.

## 2.2 Setting challenging learning goals

A good learning goal should be difficult but achievable. Therefore, the goal setting should be positioned at the position that students can achieve through their greatest efforts. On the one hand, it enables students to see that they have abilities; on the other hand, it enables them to see that their abilities have potential to be further explored. This requires teachers to understand each student's actual ability and their self-perception of ability. In the face of the goal, according to the actual ability of students to consider the goal of the absolute difficulty level and the relative difficulty level, it should set a moderate degree of difficulty learning objectives for them.

#### 2.3 Giving students the space to determine their own learning goals

Whether the goal is chosen or specified by the students themselves will have a different impact on the realization of their goal. The researchers found that allowing students to set their own learning goals increased motivation and self-regulation, presumably because self-set goals produced higher commitment, whereas individuals assigned to learning goals had lower commitment. Therefore, in mathematics teaching, students should be left with sufficient and free mathematical learning space to guide them to set their own learning goals of higher mathematics, realize their own learning goals of higher mathematics through their own efforts, and enhance their motivation of independent learning of higher mathematics.

## 3. Improving teaching methods

The key of mking students learn knowledge initiatively is to teach students the methods and strategies of learning, so that students gradually master the correct way of thinking, it should train students' ability of induction, comparison, analysis, synthesis, Abstraction and generalization, and gradually master the learning methods, so that students can truly become the master of learning.

There is a close relationship between students' academic performance and learning methods. Teachers should study not only teaching methods but also learning methods. What is more important is to teach students the Midas touch method of acquiring knowledge so that students can learn to learn. When carrying on the study method instruction, we should choose the stronger operational method what students are be able to grasp easily in the understanding student foundation, according to the student's individual characteristic, the study content, the actual ability level, in line with the principle which teaches according to one's aptitude. At the same time, we should also pay attention to the subjectivity of students, so that students take the initiative to participate in, learn to learn by themselves, take the initiative to use reasonable learning methods in accordance with certain procedures, and grasp and use knowledge independently.

#### 3.1 Discovering teaching method

Discovery teaching method is a method in which teachers provide preliminary knowledge, create a situation of positive thinking, extension and play for students, and encourage students to actively explore as inventors, discover problems, propose hypotheses and verify hypotheses, so as to acquire knowledge by themselves. Therefore, the discovery method is a good teaching method to promote students' independent learning.

Example 1. Vandermonde determinant teaching: we ask students to calculate the value of

$$\begin{vmatrix} 1 & 1 \\ x_1 & x_2 \end{vmatrix}, \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^2 \end{vmatrix}, \text{ and then guess the result of } \begin{vmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ x_1^2 & x_2^2 & \dots & x_n^2 \\ \dots & \dots & \dots & \dots \\ x_1^{n-1} & x_2^{n-1} & \dots & x_n^{n-1} \end{vmatrix}, \text{ and finally ask }$$

students to prove their guess (recursion or mathematical induction).

In this way, students are not only impressed by Vandermonde determinant, but also provide them with a way to find the truth. This example created a observation, lenovo, Abstract, generalization and mathematical process to the students which causes the students' interest in learning higher mathematics. Improving students' classroom participation and thinking of strength is conducive to the cultivation of the students' ability. In the teaching, we should pay more attention to give students' space and time of doing and thinking, students will want to learn, and learn happily and actively.

## 3.2 Variable teaching

Thinking flexibility refers to the ability to analyze specific problems on a specific basis, and when the situation changes, it have a strong strain capacity, and can use the knowledge learned flexibly to solve the problem of thinking quality.

Variable teaching refers to the teaching method that teachers use textbooks to train students' adaptability, such as asking more questions and changing frequently. It can be used to develop students' thinking flexibility.

Example 2. calculate the value of 
$$\begin{bmatrix} 0 & a & a & \cdots & a \\ a & 0 & a & \cdots & 0 \\ a & a & 0 & \cdots & a \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a & a & a & \cdots & 0 \end{bmatrix}$$

After the students solved the problem, we consciously carried out the following transformations: A. Changing 0 to 1 in the determinant. B. Changing 0 to X in the determinant. C. Changing a in

the lower position of x to 
$$-a$$
 in B. D. Changing B to 
$$\begin{vmatrix} a_1+x & a_2 & a_3 & \cdots & a_n \\ a_1 & a_2+x & a_3 & \cdots & a_n \\ & & & & & & & \\ a_1 & a_2 & a_3 & \cdots & a_n+x \end{vmatrix}$$

This series of changes activated the students' thinking and cultivated their thinking flexibility. The students showed strong interest and soon entered the state of independent learning.

## 3.3 Discussion teaching

Critical thinking refers to the thinking quality to comment on things independently, good at putting forward their own different views, do not follow others' opinions, nor self-righteous. Thinking deepness refers to the ability to delve into problems, can grasp the nature of the various attributes of things from the thinking quality.

Discussion teaching refers to the teaching method of organizing students to discuss and argue on controversial or easily confused issues, so as to clarify the issues. Obviously, it can be used to train students to think critically and deeply.

Example 3. A small elastic ball falls freely from a height of 1m. If it jumps back to half of the

same height after landing every time, will it jump forever?

After analyzing and thinking, there emerged two schools of perpetual motion theory and static theory. According to perpetual motion theory, the bounce heights of small balls form an infinite sequence  $\{h_n\}$ : 50, 25.... The infinite process is never complete and therefore does not stand still, and even if it stands still from the surface, it is the result of a very small jumping height, invisible to the naked eye.

According to the statist theory, there is an infinite decreasing geometric sequence consisting of round-trip time  $\{t_n\}$ :  $10\sqrt{2}g^{\frac{1}{2}}$ ,  $10g^{\frac{1}{2}}$ ,... (g is the acceleration of gravity calculated by the formula for the motion of a free falling body  $\mathbf{h} = \frac{1}{2}\mathbf{g}t^2$ ) since the sum of  $\mathbf{h} = \frac{1}{2}\mathbf{g}t^2$  is constant, it will not move forever.

The two sides were at loggerheads. At this point, I put forward the question whether the ball will jump forever, whether it depends on the time of jump, or whether it depends on the number of beats. After discussion, we have reached a consensus that recognition does not equal understanding. The theory of perpetual motion also proposed that if the ball can stand still, it will jump at a height of 0 meters, but 0 is not a term of  $\{h_n\}$  at all.

After further thinking, the static theory points out that the infinite bounce of a small ball can be completed within a limited time, (by infinitely decreasing geometric sequence and formula

$$s = \frac{10g^{\frac{1}{2}}}{\left(1 - \frac{\sqrt{2}}{2}\right)}$$
, and the bounce process of a small ball immediately stops after it stops, and

 $\{h_n\}$  and its limit 0 is the description of the bounce process and the static state of a small ball. 0 shouldn't be the middle term of  $\{h_n\}$ , it should be the limit of  $\{h_n\}$ .

After controversy, the students say we have not only improved the diminishing of sequence limit and infinite series, such as the formula and the understanding, is a vivid and concrete regular education, debate not only cultivate the students the scientific attitude of rigorous scholarship and grasp the essence of thinking methods, and cultivate the students' thinking critically and profundity.

In this discussion-based teaching, student not only is Structure-agency with dominant position, but also is coactee with dominant position, there is a common interest between each other, to fully mobilize enthusiasm, initiative, to fully develop creativity, thus improve the discovery, analysis and problem solving skills, open the door for autonomous learning mathematics.

## 3.4 Exploratory style teaching

Thinking broadness refers to thinking quality of thinking in multiple directions and angles, proposing multiple solutions to problems and applying a theoretical method to multiple fields. Tthinking uniqueness refers to the thinking quality of daring to be unconventional to solve problems and putting forward new opinions or new methods to solve problems. Exploratory style teaching refers to the teaching method in which teachers instruct students to explore and draw their own conclusions on a certain problem. For example, the problem of multiple solutions, it can encourage students to multi-angle thinking problems, put thinking tentacles into a variety of fields, in order to multiple solutions, and the best solution exploration teaching can be used to cultivate the broadness and uniqueness of students' thinking.

Example 4. Proving equation set

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n-1}x_{n-1} = a_{1n} \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n-1}x_{n-1} = a_{2n} \\ \dots & \dots & \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn-1}x_{n-1} = a_{nn} \end{cases}$$
 about  $x_1$   $x_2$   $\dots$   $x_{n-1}$  has solution,

then 
$$|A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0$$

Students observe, analyze and associate relevant knowledge in-depth, and propose the following proofs:

A. Direct proving method: If  $a_{nn}=0$ ,  $i=1,2,\dots,n$ , the conclusion is still true; If

 $a_{1n}, a_{2n}, \dots, a_{nn}$  are not all zero, the equation set has a solution.

- B. Reduction to absurdity (omitted)
- C. The unconventional method (obtained by teacher's inspiration): Transforming the original equation set into:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n-1}x_{n-1} - a_{1n} = 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n-1}x_{n-1} - a_{2n} = 0 \\ \dots \dots \dots \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn-1}x_{n-1} - a_{nn} = 0 \end{cases}$$

Then making  $\mathbf{x_n} = -1$ , the system of equations becomes a homogeneous linear system, and it is obvious that it has a non-zero solution at this time, so the coefficient determinant is 0, that is |A| = 0.

Explorative teaching not only obtains multiple and optimal solutions to problems, but also cultivates the broadness and uniqueness of students' thinking.

#### 4. Conclusion

The realization of independent learning is the requirement of quality education and the need of students' all-round development. Students practice through their own hands, and think actively by using their own brains. Discover and innovation could give full play to students' autonomy, initiative and enthusiasm, and really play the main role of students. The acquisition of independent learning ability is a process of internalizing external learning skills into one's own ability. The cultivation of independent learning ability should be carried out in independent learning activities. There are two necessary preconditions for the implementation of independent learning, that is, learners have the ability of independent learning, and the school provides the space for independent learning. The former is a subjective condition, while the latter is an objective condition.

#### References

- [1] Pang Weiguo, Autonomous learning: principles and strategies for teaching and learning, East China Normal University Press, 2003.
- [2] Pei Hongbing, Zhang Guochu, Ye Jun, On the training mode of mathematics autonomous learning ability, Journal of Mathematics Education, vol.2, 2005.